Review Problems for Exam 3

Problem 1. Let $v = x^2 \sin y + ye^{xy}$, $x = s + 2t$, $y = st$. Use the chain rule to find $\partial v/\partial s$ and $\partial v/\partial t$ when $s = 0$ and $t = 1$.

Problem 2. A manufacturer has modeled its yearly production function P (the value of its entire production in millions of dollars) as the function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the amount of invested capital (in millions of dollars). Suppose that when $L = 30$ and $K = 8$, the labor force is decreasing at a rate of 2000 labor hours per year, and capital is increasing at a rate of 500,000 dollars per year. Find the rate of change of production.

Problem 3. Let $f(x, y) = x^2 y + \sqrt{y}$.

a. Find $\nabla f(x, y)$.
b. i. Find the directional derivative of f at $(2, 1)$ in the direction towards the point $(5, 3)$.
   ii. Explain what this value means.
c. What is the maximum rate of change of f at $(2, 1)$?
d. In which direction does the maximum rate of change of f occur?

Problem 4. Find (a) an equation of the tangent plane and (b) parametric equations of the normal line to the surface $xy + yz + zx = 5$ at the point $(1, 2, 1)$.

Problem 5. Find the local maxima, local minima, and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

Problem 6. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (twice the sum of its width and height) is at most 108 in. Suppose you want to find dimensions of the package with the largest volume that can be mailed.

a. Write an optimization model for your problem (i.e. What is the function you are maximizing/minimizing? What are the constraints?)
b. Solve this optimization model using your method of choice.